Thermally and mechanically induced residual strain and strain tolerance of critical current in stainless steel-laminated Bi2223/Ag/Ag alloy composite superconductors

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Abstract
It is known that the tensile strain tolerance of the critical current of Bi2223 composite superconductors is improved when the compressive residual strain of Bi2223 filaments in the current transport direction (sample length direction) is enhanced. In the present work, the thermally and mechanically induced residual strain accumulation process of a stainless steel-laminated Bi2223/Ag/Ag alloy superconducting composite tape fabricated at American Superconductor Corporation was studied. A calculation procedure is presented for a description of the strain change of the constituents (Bi2223, Ag, Ag alloy and stainless steel) during cooling and heating, and during lamination, followed by the stress relaxation. As the input values, the mechanical property values of the constituents and the residual strain of Bi2223 filaments in the laminated composite tape at room temperature, measured with x-rays, were used. Such an approach revealed the change of residual strain of each constituent in the thermal and mechanical process in the laminated composite tape. Then, from a comparison of the residual strain accumulation in the laminated tape with that in the insert tape, the effects of the stainless steel lamination on the enhancement of the compressive residual strain of Bi2223 filaments and the improvement of the tensile strain tolerance and strain tolerance window of critical current at 77 K of the composite tape are discussed.

List of strain variables

| Δε1,IT, Δε1,SS: | Increment of strain of insert tape (IT) and stainless steel (SS) due to pre-load for lamination |
| Δε2,IT, Δε2,SS: | Change of elastic strain of insert tape (IT) and stainless steel (SS) in the strain range from elastic state to compressive yielding of Ag in the stress relaxation process after lamination |

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1. Introduction

When a Bi2223/Ag/Ag alloy composite tape is cooled from the heat-treatment temperature to the cryogenic one, a compressive residual strain is accumulated in the Bi2223 filaments in the current transporting direction due to the difference in coefficient of thermal expansion among the constituents (Bi2223, Ag and Ag alloy sheath) [1–15]. It is known that such a compressive residual strain of Bi2223 filaments contributes largely to the tensile strain tolerance of the critical current of Bi2223/Ag/Ag alloy composite tape [1–28].

The compressive residual strain of Bi2223 filaments is increased by reinforcing the Bi2223/Ag/Ag alloy composite tape with materials having high modulus and low coefficient of thermal expansion [1–6]. It has been demonstrated that, when reinforced with stainless steel or molybdenum, the tensile strain tolerance of the critical current, and the size of the strain tolerance window, defined as the axial strain range from compression through tension in which the critical current remains above 95% of its original level, and the strength of the composite tape are improved [1–6].

In a laminated tape, the residual strain of each constituent (Bi2223, Ag, Ag alloy and stainless steel) is composed of (a) the mechanically induced strain by lamination and (b) the thermally induced strain by temperature change, due to the following mechanism.

(a) Mechanically induced strain by lamination: in the reinforcing process, the reinforcing material (strip) is solder-laminated on the insert Bi2223/Ag/Ag alloy composite tape at elevated temperatures and at selected pre-tension levels in the strips and the insert composite tape [1–4]. When the externally imposed pre-tension is removed after lamination, the mechanically coupled composite relaxes axially to equilibrium where the constituents are under residual axial stress but the net force is zero [1]. As the tensile pre-load applied on the strips is higher than that on the insert composite tape, compressive strain is given to Bi2223, Ag and Ag alloy in the insert tape after the load relaxation.

(b) Thermally induced strain by temperature change: the composite tape is exposed in the following temperature change. First the insert tape is cooled down from the heat-treatment temperature to room temperature and then it is heated to the lamination temperature. After the lamination, the laminated composite is cooled down to room temperature and then to cryogenic temperature for testing. After the test, it is heated to room temperature. Under such a temperature change, strain is induced in each constituent due to the difference in coefficient of thermal expansion between the constituents.

The total strain of the elastically deforming constituents (Bi2223, Ag alloy and stainless steel) is the sum of mechanically and thermally induced strains. Ag behaves elastically and plastically depending on the stress situation, since the yield strain is low (0.02% [27]). Thus the total strain of Ag is not the sum. Such an elastic–plastic behavior shall be incorporated for estimation.

The aim of the present work is to describe the accumulation process of the mechanically and thermally induced strains of each constituent (Bi2223, Ag, Ag alloy and stainless steel) in the stainless steel-laminated composite tape by elastic–plastic mechanics and to discuss the effects of stainless steel lamination on the residual strain accumulation process and critical current at 77 K. The present work has the following content.

(1) The mechanically and thermally induced residual strain of Bi2223 at 77 K in the laminated tape has been analyzed by Otto et al [1] and Vaccio et al [6]. In their analysis, the elastic–plastic behavior of Ag has, however, not been incorporated. In the present work, their approach was extended so as to express the thermo-mechanical behavior of the constituents by incorporating the elastic–plastic behavior of Ag, as will be shown in sections 3.1 and 3.2.
Figure 1. Polished cross section of the supplied stainless steel-laminated composite tape.

(2) A laminated composite tape fabricated by American Superconductor Corporation (AMSC), in which the high critical current type Bi2223/Ag/Ag alloy composite tape was used as the insert tape, was studied in the present work. The stress–strain behavior and residual strain accumulation process of the insert tape have been studied in previous work [27]. The results obtained for the insert tape were used in the analysis of the present work, as will be shown in sections 3.2 and 3.4. Also the residual strain of Bi2223 in the laminated composite tape at room temperature was measured in the present work at SPring 8, Japan. The measured value was used to estimate the effectively acting pre-load on stainless steel in the lamination process, as will be shown in section 3.3.

(3) From the comparison of the result on the residual strain change in the mechanical and thermal process for the laminated composite tape obtained in the present work with that for the insert tape obtained in the previous work [27], the lamination effect on the residual strain accumulation behavior was extracted, as will be shown in section 3.4.

(4) From the comparison of the critical current-applied tensile strain relation of the laminated tape at 77 K with that of the insert tape, the effect of lamination on the tensile strain tolerance and strain tolerance window of the critical current is discussed, as will be shown in section 3.5.

2. Procedure for experiment and analysis

2.1. Samples

The stainless steel-laminated composite tape, consisting of an insert tape, stainless steel and solder, was supplied by AMSC. In this composite tape, a high critical current \( I_c \) tape is used as the insert tape, whose residual strain accumulation process has been shown in our previous work [27]. The polished cross-sectional areas of the insert tape, stainless steel and solder are listed in table 1. The lamination temperature, at which the insert tape was laminated with stainless steel (SS), has been reported to be 453 K [1]. The supplied composite tape had been cooled down from the heat-treatment temperature (TH) to room temperature, heated to lamination temperature (453 K), cooled down to 77 K through room temperature for pre-checking of the critical current and then heated to room temperature, as schematically shown in figure 2. Due to such a thermal history, the Ag had been yielded in compression at room temperature, as will be shown in section 3.4.

Figure 2. Thermal and mechanical history of the composite tape used in the present work.

2.2. Measurement of the residual strain of Bi2223 filaments at room temperature

The residual strain in the length direction of Bi2223 filaments in the composite tape at room temperature was measured at the beam line 46XU of a synchrotron-radiation facility, SPring 8, Japan. The procedure is the same as that in our previous work [27]. As a stress-free reference sample, the bare Bi2223 filaments, extracted from the composite tape by etching away the Ag and Ag alloy with a NH\(_4\)OH/H\(_2\)O\(_2\) solution, was used. Tensile strain \( \varepsilon_c \) was given to the samples up to 0.1% in a step of around 0.03%. The peak positions of the (220) planes were measured for the strained Bi2223 filaments in the composite as well as for the strain-free extracted Bi2223 ones. From the difference in the peak position between the strained and strain-free Bi2223 filaments, the strain of Bi2223 filaments \( \varepsilon_{Bi, r} \), corresponding to \( \varepsilon_{c,T} = 0\% \), was estimated by applying the least square method to the measured relation between the strain of Bi2223 filaments in the composite and applied strain on the composite \( \varepsilon_{c,T} \).

2.3. Key temperatures for analysis

The key temperatures in the thermo-mechanical history of the insert and laminated composite tapes are schematically shown in figure 3. The insert tape is heat-treated at TH and is cooled down to room temperature. During cooling, the strain accumulation starts practically at T0 since the residual strains are released by the creep behavior of Ag/Ag alloy in the high temperature region. The T0-value of the insert tape has been estimated in our previous work [27] to be 563 K in terms of the
Ag in the present insert tape has a low yield strain (0.02% [27]), in Ag and Ag alloy and compressive strain in Bi2223. As the expansion than Bi2223, tensile strain is induced during cooling range. property values of the constituents in the relevant temperature range.

As Ag and Ag alloy have higher coefficient of thermal expansion than Bi2223, tensile strain is induced during cooling in Ag and Ag alloy and compressive strain in Bi2223. As the Ag in the present insert tape has a low yield strain (0.02% [27]), it is yielded in tension in and compression during cooling and heating, respectively, when the exerted strain reaches the yield strain. In the insert tape, the change of zero strain to tensile (and compressive) yielding state is caused by the temperature difference 69 K and the change of tensile to compressive (and vice versa) yielding by the temperature difference 138 K, as will be shown in section 3.2. Thus, during cooling from T0 (563 K) to room temperature 293 K (RT(1) in figure 3), Ag is yielded in tension at T1 (= 563 − 69 = 494 K) before the temperature reaches room temperature. Thus at room temperature, Ag has been yielded in tension. Then, from the room temperature RT(1), the tape is heated to T3 for lamination. During heating, compressive stress is exerted on Ag, due to which Ag is yielded in compression at T2 (= 293 + 138 = 431 K).

At the lamination temperature T3 (453 K), Ag has been yielded in compression. Such a situation is denoted as T3, 0 in figure 3. The insert tape is pre-loaded in tension and is solder-laminated with the pre-loaded stainless steel (T3, 1). In the pre-tensile loading of the insert tape, Ag, which has been yielded in compression at T3, 0, deforms elastically and tensile strain is added, but the strain of Ag does not exceed the tensile yield strain due to the low pre-load on the insert tape (4.5 N [1]), as will be shown later. After the lamination, the pre-loads are relaxed. In the load relaxation process, the Ag is compressed first elastically until Ag is yielded in compression at T3, 2. Then Ag deforms plastically in compression until the load is completely relaxed at T3, 3.

After the load relaxation, the laminated tape is cooled down from T3, 3 to room temperature RT(2) and then from
RT(2) to 77 K(1). On the way, Ag that has been yielded in compression at T3, 3 is yielded in tension at T4. Such a change from tensile to compressive (and vice versa) yielding is caused by the temperature difference 200 K in the laminated tape, as will be shown in section 3.2. At 77 K(1), Ag is in a tensile yielded state. The critical current of the tape was pre-checked at AMSC at 77 K(1). Then the tape was heated from 77 K(1) to room temperature RT(3). On the way, as compressive stress is exerted on Ag, Ag is yielded in compression at T5. Thus at RT(3), Ag is in a compressively yielded state, while at RT(2) before the cooling down to 77 K, Ag is in a tensile yielded state. At room temperature RT(3), the residual strain of Bi2223 filaments was measured by x-ray diffraction, as shown later in section 3.3. When the laminated tape is again cooled down from room temperature RT(3) to 77 K(2) for measurement of critical current, the Ag, which has been yielded in compression at RT(3), is yielded in tension at T6. Thus at 77 K(2), Ag is yielded in tension, which is the same situation as at 77 K(1). When the tape is heated from 77 K(2) to room temperature RT(4), Ag comes to be yielded in compression at T7 on the way. Thus Ag is in a compressively yielded state at RT(4), which is the same situation as at RT(3), but not the same situation as at RT(1) at which Ag is in a tensile yielded state. As the stress state at RT(3) is the same as that at RT(4) and also the stress state at 77 K(1) is the same as that at 77 (K), T7 is equal to T5, as will be shown in sections 3.3 and 3.4.

The mechanical strain is induced through the application of pre-loads to the insert tape and stainless steel, lamination and the subsequent load relaxation. Taking the behavior at T3 (T3, 0 to T3, 3) stated above into consideration, the mechanically induced strain in each constituent (Bi2223, Ag, Ag alloy and stainless steel) will be formulated in section 3.1, which will be used in section 3.3 for estimation of effectively acting pre-load on stainless steel. The temperature change-induced thermal strain will be formulated in section 3.2, which will be used in section 3.4 for the calculation of the change of residual strain of each constituent as a function of temperature.

2.4. Input values for analysis

For analysis of the strain accumulation in the constituents, the property values listed in table 1 were used. As the strain is accumulated in the insert tape and the laminated tape before and after the lamination, respectively (figures 2 and 3), the property values for both insert and laminated tapes are included. They were obtained in the following manner.

(a) The cross-sectional area $A_i$ ($i = \text{laminated tape, insert tape, stainless steel and solder}$) was measured from the SEM image of the cross section shown in figure 1.

(b) The volume fraction $V_{i,LT}$ of the constituents in the laminated tape ($i = \text{insert tape, stainless steel and solder}$, and the subscript LT refers to the laminated tape) was calculated from the cross-sectional area obtained in (a).

(c) The term, Young’s modulus $\times$ volume fraction, $E_i V_{i,IT}$, of the constituents in the insert tape ($i = \text{Ag, Ag alloy and Bi2223}$, and IT refers to the insert tape) were taken from [27].

(d) The overall Young’s moduli of the insert tape in region I, where Ag, Ag alloy and Bi2223 deform elastically, and that in region II, where Ag deforms plastically and Ag alloy and Bi2223 deform elastically, were taken from [27].

(e) The Young’s modulus of the stainless steel was taken from [1].

(f) The term, Young’s modulus $\times$ volume fraction, $E_i V_{i,LT}$, of the constituents in the laminated tape ($i = \text{Ag, Ag alloy, Bi2223, stainless steel and solder}$) were calculated from (b), (c) and (e) and $E_{\text{Solder}} = 24 \ \text{GPa}$ [1].

(g) The overall Young’s moduli of the laminated tape in region I ($E_{\text{L,LT}}$) where all constituents (Bi2223, Ag, Ag alloy, stainless steel and solder) deform elastically and in region II ($E_{\text{II,LT}}$), where Ag deform plastically and other constituents deform elastically, were calculated by the rule of mixtures as follows.

For region I, the overall Young’s modulus, $E_{\text{L,LT}}$, is given by

$$E_{\text{L,LT}} = E_{\text{Ag}} V_{\text{Ag,LT}} + E_{\text{Alloy}} V_{\text{Alloy,LT}} + E_{\text{Bi}} V_{\text{Bi,LT}}$$

$$+ E_{\text{SS}} V_{\text{SS,LT}} + E_{\text{Solder}} V_{\text{Solder,LT}}$$

(Region I) (1)

$$E_{\text{II,LT}} = E_{\text{Alloy}} V_{\text{Alloy,LT}} + E_{\text{Bi}} V_{\text{Bi,LT}} + E_{\text{SS}} V_{\text{SS,LT}}$$

$$+ E_{\text{Solder}} V_{\text{Solder,LT}}$$

(Region II). (2)

Substituting the $E_i V_{i,LT}$ ($i = \text{Ag, Ag alloy, Bi2223, stainless steel and solder}$) values obtained in (f), the $E_{\text{L,LT}}$ and $E_{\text{II,LT}}$ were calculated.

(h) The yield strain of Ag was taken from [27].

(i) The coefficients of thermal expansion of Ag ($\alpha_{\text{Ag}}$), Ag alloy ($\alpha_{\text{Alloy}}$) and Bi2223 ($\alpha_{\text{Bi}}$) were taken from [27]. The coefficient for stainless steel ($\alpha_{\text{SS}}$) was taken from [29].

(j) The overall coefficient of thermal expansion of insert tape was taken from [27].

(k) The overall coefficients of thermal expansion of the laminated tape for regions I ($\alpha_{\text{L,LT}}$) and II ($\alpha_{\text{II,LT}}$) were calculated by equations (17) and (18) shown later.

3. Results and discussion

3.1. Formulation of mechanically induced residual strain in the lamination, followed by the load relaxation process

The residual strain of Bi2223 filaments at room temperature $\varepsilon_{\text{Bi,RT}}$ (RT) in the stainless steel-laminated composite tape was measured by the x-ray diffraction method at RT(3); see figure 3. It is given by the sum of the mechanical strain $\varepsilon_{\text{Bi,M}}$ arising from the lamination, followed by load relaxation, and the thermally accumulated strain at RT(3) $\varepsilon_{\text{Bi,T}}$ (RT) arising from the difference in thermal expansion among the constituents [1, 6];

$$\varepsilon_{\text{Bi,RT}} = \varepsilon_{\text{Bi,M}} + \varepsilon_{\text{Bi,T}} (\text{RT})$$

(3)

In the present work, $\varepsilon_{\text{Bi,M}}$ and $\varepsilon_{\text{Bi,T}}$ (RT) were derived as follows. The Bi2223, stainless steel and Ag alloy were treated as the elastic body. As Ag is soft [27], it was treated as an elastic-perfect plastic body with a yield strain $\varepsilon_{\text{Ag,Y}} = 0.02\%$, as in our previous work [27].

In this section, the change of mechanical strain of insert tape and stainless steel due to the lamination is analyzed. In
the lamination process, the insert tape and stainless steel are heated to the lamination temperature T3 (453 K) and are pulled in tension up to the loads $F_{0, IT}$ and $F_{0, SS}$, respectively. Under the pre-loaded condition, they are bonded together with solder. After the bonding, the total load $F_{0, SS} + F_{0, IT}$ is relaxed until the total load of the laminated tape reaches zero in the filament axis direction. The change of the strain of constituents is calculated by dividing the process into the following stages.

Stage 0 (T3, 0 in figure 3):

The insert tape and stainless steel are heated to 453 K (T3). As the yield strain of Ag is low (0.02%), Ag in the insert tape has been yielded in tension at room temperature (RT(1)) during cooling from the heat-treatment temperature [27]. When the insert tape is heated to 453 K (T3), the Ag is yielded in compression on the way, as will be shown later in sections 3.2 and 3.4.

Stage 1 (T3, 0 → T3, 1 in figure 3):

The insert tape and stainless steel are pulled in tension up to the loads $F_{0, IT}$ and $F_{0, SS}$, respectively, and are bonded together with solder at 453 K. As Ag in the insert tape is yielded in compression, it deforms elastically upon tensile loading. Thus the increment of the strain of the insert tape $\Delta \varepsilon_{1, IT}$ is given by

$$\Delta \varepsilon_{1, IT} = F_{0, IT} / (E_{IT} A_{IT}), \quad (4)$$

where $E_{IT}$ is the overall Young’s modulus in region I and $A_{IT}$ is the cross-sectional area of the insert tape (table 1). $F_{0, IT}$ has been reported to be 4.5 N [1]. Substituting these values into equation (4), we have $\Delta \varepsilon_{1, IT} = 0.006\%$, which is lower than the yield strain 0.02% of Ag. Thus Ag in the loaded insert tape is in the elastic range. The increment of strain $\Delta \varepsilon_{1, SS}$ of the stainless steel due to the applied load $F_{0, SS}$ is given by

$$\Delta \varepsilon_{1, SS} = F_{0, SS} / (E_{SS} A_{SS}), \quad (5)$$

where $E_{SS}$ and $A_{SS}$ are the Young’s modulus and cross-sectional area of the stainless steel, respectively (table 1).

Stage 2 (T3, 1 → T3, 2 in figure 3):

After the lamination, the total load $F_{0, IT} + F_{0, SS}$ is relaxed. In the relaxation process, the stainless steel that is more strained by the higher tensile pre-load (71.4 N as shown later) shrinks more than the insert tape that is less strained by the lower tensile pre-load (4.5 N). Accordingly, the compressive and tensile loads are given to the insert tape and stainless steel, respectively, after the load relaxation. In the initial stage of relaxation, Ag in the insert tape, which is in the elastic state, deforms elastically until Ag is yielded in compression. The change of the elastic strain $\Delta \varepsilon_{2, IT}$ of the insert tape and that of stainless steel $\Delta \varepsilon_{2, SS}$, corresponding to the strain range from elastic state to compressive yielding of Ag, are given by

$$\Delta \varepsilon_{2, IT} = \Delta \varepsilon_{2, SS} = -\Delta (F_{0, IT} + F_{0, SS})_2 / (E_{IT} A_{IT}). \quad (6)$$

where $-\Delta (F_{0, IT} + F_{0, SS})_2$ is the change of the load. The strain range $\Delta \varepsilon_{2, IT} = \Delta \varepsilon_{2, SS}$ is equal to $-\Delta \varepsilon_{1, IT}$ (equation (4)), which is expressed as

$$\Delta \varepsilon_{2, IT} = \Delta \varepsilon_{2, SS} = -\Delta (F_{0, IT} + F_{0, SS})_2 / (E_{IT} A_{IT}) = -\Delta \varepsilon_{1, IT}. \quad (7)$$

Substituting the known values of $E_{IT}$ and $A_{IT}$ listed in table I and $-\Delta \varepsilon_{1, IT} = -0.006\%$ mentioned above into equation (7), $-\Delta (F_{0, IT} + F_{0, SS})_2$ is calculated to be $-8.04$ N. This means that the total applied load $F_{0, IT} + F_{0, SS}$ is consumed by 8.04 N and the remaining load is $F_{0, IT} + F_{0, SS} - \Delta (F_{0, IT} + F_{0, SS})_2$.

Stage 3 (T3, 2 → T3, 3 in figure 3):

After the stress of Ag reaches its compressive yield stress, Ag deforms plastically. The change of strain of the insert tape, $\Delta \varepsilon_{3, IT}$, and stainless steel, $\Delta \varepsilon_{3, SS}$, from the onset of the compressive yielding of Ag to the final stress state at which the stress reaches the equilibrium state (total load of the composite is zero) is given by

$$\Delta \varepsilon_{3, IT} = \Delta \varepsilon_{3, SS} = -(F_{0, IT} + F_{0, SS})_2 / (E_{IT} A_{IT}). \quad (8)$$

Thus, the mechanical strain of the insert tape $\varepsilon_{IT,M}$ and stainless steel $\varepsilon_{SS,M}$ induced by the lamination and subsequent stress relaxation is given by

$$\varepsilon_{IT,M} = \Delta \varepsilon_{1, IT} + \Delta \varepsilon_{2, IT} + \Delta \varepsilon_{3, IT} = -(F_{0, IT} + F_{0, SS})_2 / (E_{IT} A_{IT}) \quad (9)$$

$$\varepsilon_{SS,M} = \Delta \varepsilon_{1, SS} + \Delta \varepsilon_{2, SS} + \Delta \varepsilon_{3, SS} = -(F_{0, IT} + F_{0, SS})_2 / (E_{SS} A_{SS})\quad (10)$$

The mechanically induced strain in Bi2223, $\varepsilon_{IT,M}$ and that in Ag alloy, $\varepsilon_{Alloy,M}$, are the same as that of $\varepsilon_{IT,M}$. As Ag is yielded in compression after the load relaxation, the elastic strain of Ag is $-\varepsilon_{Ag,y}$. The unknown value of $F_{0, SS}$ will be obtained later in section 3.3 from the comparison of the calculation result with the residual strain of Bi2223 measured with x-rays.

3.2. Formulation of thermally induced strain during cooling and heating

The thermally induced residual strain of Bi2223 varies in the insert tape until it is laminated with stainless steel, and it varies also in the laminated tape during cooling from the lamination temperature (T3). Thus the thermal history of the insert tape until the lamination, and that of the laminated tape after the lamination (figure 3), shall be incorporated for calculation of the change of thermal strain.

The necessary formulae and parameters for thermal stress analysis for the insert and laminated tapes are given as follows.

3.2.1. Formulation for calculation of thermally induced residual strain change before lamination. The thermal history and the change of thermally induced strain of Bi2223 in the insert tape have been studied in our previous paper [27]. Using the result for the insert tape, the coefficient of the overall thermal expansion of the insert tape for regions I (all constituents deform elastically) and II (Ag deforms plastically and the other constituents deform elastically) is given by

$$\alpha_{IT} = (\alpha_{Bi} E_{Bi} V_{Bi, IT} + \alpha_{Ag} E_{Ag} V_{Ag, IT} + \alpha_{Alloy} E_{Alloy} V_{Alloy, IT}) / E_{IT}$$

\[(\text{Region I})\]

$$\alpha_{II} = (\alpha_{Bi} E_{Bi} V_{Bi, II} + \alpha_{Alloy} E_{Alloy} V_{Alloy, II}) / E_{II}$$

\[(\text{Region II})\]
In region I, the thermally induced elastic strain of the constituents for the change in temperature \(\Delta T\) is given by

\[
\Delta \varepsilon_{\text{Bi,IT}} = (\alpha_{\text{Bi}} - \alpha_{\text{Bi}}) \Delta T, \\
\Delta \varepsilon_{\text{Ag,IT}} = (\alpha_{\text{Ag}} - \alpha_{\text{Ag}}) \Delta T, \\
\Delta \varepsilon_{\text{Alloy,IT}} = (\alpha_{\text{Alloy}} - \alpha_{\text{Alloy}}) \Delta T.
\] (13)

In region II, the thermally induced elastic strains of Bi2223 and Ag alloy are given by

\[
\Delta \varepsilon_{\text{Bi,II}} = (\alpha_{\text{Bi}} - \alpha_{\text{Bi}}) \Delta T, \\
\Delta \varepsilon_{\text{Alloy,II}} = (\alpha_{\text{Alloy}} - \alpha_{\text{Alloy}}) \Delta T.
\] (14)

The total strain of Ag under plastic deformation (region II) is the sum of the elastic strain \(\varepsilon_{\text{Ag,elastic}}\) and plastic strain \(\varepsilon_{\text{Ag,plastic}}\). Under the approximation of Ag as an elastic-perfect plastic body, only the elastic strain is concerned with the stress carrying capacity of Ag, and the stress of Ag, \(\sigma_{\text{Ag}}\), is given by

\[
\varepsilon_{\text{Ag,elastic}} E_{\text{Ag}}. \quad \text{The elastic component of residual strain of Ag, } \\
\varepsilon_{\text{Ag,IT,elastic}} \text{ in region II is given by [27]}
\]

\[
\varepsilon_{\text{Ag,IT,elastic}} = +\alpha_{\text{Ag,y}} \quad \text{when Ag deforms plastically is tension}
\]

\[
\varepsilon_{\text{Ag,IT,elastic}} = -\alpha_{\text{Ag,y}} \quad \text{when Ag deforms plastically is compression.}
\] (15) (16)

### 3.2.2. Formulation for calculation of thermally induced residual strain change after lamination.

After lamination, Bi2223 filaments exist in the laminated tape. The Young’s modulus of the laminated tape \(E_{\text{LTT}}\) for elastically deforming Ag (region I) and \(E_{\text{ILT}}\) for plastically deforming Ag (region II) are listed in table 1. The coefficient of thermal expansion of the laminated tape is given by

\[
\alpha_{\text{LT}} = (\alpha_{\text{Bi}} E_{\text{Bi}} V_{\text{Bi,LT}} + \alpha_{\text{Ag}} E_{\text{Ag}} V_{\text{Ag,LT}} + \alpha_{\text{Alloy}} E_{\text{Alloy}} V_{\text{Alloy,LT}} + \alpha_{\text{SS}} E_{\text{SS}} V_{\text{SS,LT}} + \alpha_{\text{Solder}} E_{\text{Solder}} V_{\text{Solder,LT}})/E_{\text{LTT}}
\] (Region I)

\[
\alpha_{\text{ILT}} = (\alpha_{\text{Bi}} E_{\text{Bi}} V_{\text{Bi,ILT}} + \alpha_{\text{Alloy}} E_{\text{Alloy}} V_{\text{Alloy,ILT}} + \alpha_{\text{SS}} E_{\text{SS}} V_{\text{SS,ILT}} + \alpha_{\text{Solder}} E_{\text{Solder}} V_{\text{Solder,LT}})/E_{\text{ILT}}
\] (Region II).

In region I, the thermally induced elastic strain change of Bi2223, Ag, Ag alloy and stainless steel for the temperature range \(\Delta T\) is given by

\[
\Delta \varepsilon_{\text{Bi,LT}} = (\alpha_{\text{Bi}} - \alpha_{\text{Bi}}) \Delta T, \\
\Delta \varepsilon_{\text{Ag,LT}} = (\alpha_{\text{Ag}} - \alpha_{\text{Ag}}) \Delta T, \\
\Delta \varepsilon_{\text{Alloy,LT}} = (\alpha_{\text{Alloy}} - \alpha_{\text{Alloy}}) \Delta T, \\
\Delta \varepsilon_{\text{SS,LT}} = (\alpha_{\text{SS}} - \alpha_{\text{SS}}) \Delta T.
\] (19)

In region II, the thermally induced elastic strain for Bi2223, Ag alloy and stainless steel is given by

\[
\Delta \varepsilon_{\text{Bi,II}} = (\alpha_{\text{Bi}} - \alpha_{\text{Bi}}) \Delta T, \\
\Delta \varepsilon_{\text{Alloy,II}} = (\alpha_{\text{Alloy}} - \alpha_{\text{Alloy}}) \Delta T, \\
\Delta \varepsilon_{\text{SS,II}} = (\alpha_{\text{SS}} - \alpha_{\text{SS}}) \Delta T.
\] (20)

and the elastic component of residual strain of Ag, \(\varepsilon_{\text{Ag,LT,elastic}}\) in region II is given in a similar manner to those for the insert tape.

\[
\varepsilon_{\text{Ag,LT,elastic}} = +\alpha_{\text{Ag,y}} \quad \text{when Ag deforms plastically is tension}
\]

\[
\varepsilon_{\text{Ag,LT,elastic}} = -\alpha_{\text{Ag,y}} \quad \text{when Ag deforms plastically is compression.}
\] (21) (22)

### 3.2.3. Calculation of thermally induced strain change.

The thermally induced residual strain change of the constituents is calculated by equations (11)–(22) as a function of temperature, as follows. The necessary values for calculation are listed in table 1.

**Step 0**

The thermal strain is practically accumulated in the insert tape below the temperature \(T_0\) (563 K) in terms of the material property values in the relevant temperature range (77–600 K), as has been shown in our previous paper [27].

**Step 1 (T0 \(\rightarrow\) T1)**

During cooling from \(T_0\) (563 K), Ag in the insert tape deforms elastically until the temperature reaches \(T_1\) and then plastically in the temperature range from \(T_1\) to room temperature (\(RT = 293 K\)). \(T_1\) satisfies

\[
(\alpha_{\text{1,}T} - \alpha_{\text{Ag}})(T_1 - T_0) = \varepsilon_{\text{Ag,y}}.
\] (23)

\(T_1\) was calculated to be 494 K. This means that the change from zero strain to yielding state of Ag is caused by the temperature difference 69 K. The change of strain of each constituent from \(T_0\) to \(T_1\), \(\Delta \varepsilon_{i,T_0 \rightarrow T_1}\) \((i = \text{Bi2223, Ag and Ag alloy})\), is calculated by equation (13). The strain of Ag varies from zero at \(T_0\) to \(+\varepsilon_{\text{Ag,y}}\) at \(T_1\).

**Step 2 (T1 \(\rightarrow\) RT(1))**

Upon further cooling of the insert tape from \(T_1\) (495 K) to room temperature 293 K (RT(1) in figure 3), Ag deforms plastically in tension. The change of strain of each constituent from \(T_1\) to RT, \(\Delta \varepsilon_{i,T_1 \rightarrow RT}\) \((i = \text{Bi2223 and Ag alloy})\), is calculated by equation (14). The elastic strain of Ag does not change from \(T_1\) to RT(1), being equal to \(+\varepsilon_{\text{Ag,y}}\).

**Step 3 (RT(1) \(\rightarrow\) T2)**

Then the insert tape is heated to the lamination temperature \(T_3 = 453 K\) from RT(1). In this process, as the compressive thermal stress is imposed on Ag, Ag deforms first elastically up to the temperature \(T_2\) at which Ag is yielded in compression, and then deforms plastically in compression up to the lamination temperature \(T_3\). The temperature \(T_2\) satisfies

\[
(\alpha_{\text{1,}T} - \alpha_{\text{Ag}})(T_2 - RT) = -2\varepsilon_{\text{Ag,y}}.
\] (24)

\(T_2\) was calculated to be 431 K. This means that the change of tensile to compressive yielding state (and also the change of compressive to tensile yielding state) of Ag is caused by the temperature difference 138 K in the insert tape. The change of strain of each constituent from RT to \(T_2\), \(\Delta \varepsilon_{i,RT \rightarrow T_2}\) \((i = \text{Bi2223, Ag and Ag alloy})\), is calculated by equation (13). The elastic strain of Ag changes from \(+\varepsilon_{\text{Ag,y}}\) at RT(1) to \(-\varepsilon_{\text{Ag,y}}\) at \(T_2\).
Step 4 (T2 → T3)

Upon further heating of the insert tape from T2 (431 K) to T3 (lamination temperature: 453 K), Ag deforms plastically in compression. The change of strain of each constituent from T2 to T3, \( \Delta \varepsilon_{\text{f}, \text{t}2-\text{t}3} \) (\( i = \text{Bi2223 and Ag alloy} \)), is calculated by equation (14). The elastic strain of Ag remains at \( -\varepsilon_{\text{Ag},y} \).

Step 5 (T3 → RT(2))

At T3 (453 K), the insert tape and stainless steel are assembled. As will be shown in section 3.3, Ag is yielded in compression after the load relaxation. Thus, upon cooling from T3, as tensile stress is thermally imposed on Ag, Ag behaves elastically until the temperature reaches T4 at which Ag is yielded in tension. The temperature T4 satisfies

\[
(\alpha_{\text{LT}} - \alpha_{\text{Ag}})(T4 - T3) = 2\varepsilon_{\text{Ag},y}.
\]

(25)

T4 was calculated to be 253 K. This means that the change of compressive to tensile yielding state (and also the change of tensile to compressive yielding state) of Ag in the laminated tape is caused by the temperature difference 200 K, which is larger than 138 K in the insert tape. As RT(293 K) > T4, Ag is in the elastic state at RT(2). The change of strain of each constituent from T3 to RT(2), \( \Delta \varepsilon_{\text{f}, \text{t}3-\text{RT}(2)} \) (\( i = \text{Bi2223, Ag, Ag alloy and stainless steel} \)), is calculated by equation (19). The elastic strain of Ag changes from \( -\varepsilon_{\text{Ag},y} \) to \( -\varepsilon_{\text{Ag},y} + \Delta \varepsilon_{\text{Ag},y} \) at T3 and \( -\varepsilon_{\text{Ag},y} + \Delta \varepsilon_{\text{Ag},y} \) at RT(2).

Step 6 (RT(2) → T4)

As shown in step 5, T4 was 253 K. Upon further cooling of the laminated tape from RT(2) to T4, Ag deforms elastically in tension. The change of strain of each constituent from RT(2) to T4, \( \Delta \varepsilon_{\text{f}, \text{t}4-\text{RT}(2)} \) (\( i = \text{Bi2223, Ag, Ag alloy and stainless steel} \)), is calculated by equation (19). The elastic strain of Ag becomes \( +\varepsilon_{\text{Ag},y} \) at T4.

Step 7 (T4 → 77 K(1))

During further cooling of the laminated tape from T4 (253 K) to 77 K(1), Ag deforms plastically in tension. The change of strain of each constituent from T4 to 77 K, \( \Delta \varepsilon_{\text{f}, \text{t}4-\text{77 K}} \) (\( i = \text{Bi2223, Ag alloy and stainless steel} \)), is calculated by equation (20). The elastic strain of Ag remains at \( +\varepsilon_{\text{Ag},y} \).

Step 8 (77 K(1) → T5)

At 77 K, Ag is yielded in tension. When the laminated tape is heated from 77 K(1), compressive stress is imposed on Ag. Thus Ag behaves elastically up to T5 at which Ag is yielded in compression. The temperature T5, satisfying \( (\alpha_{\text{LT}} - \alpha_{\text{Ag}})(T5 - 77 K) = -2\varepsilon_{\text{Ag},y} \), was calculated to be 277 K. The change of strain of each constituent from 77 K(1) to T5, \( \Delta \varepsilon_{\text{f}, \text{t}77 \text{K}-\text{t}5} \) (\( i = \text{Bi2223, Ag, Ag alloy and stainless steel} \)), is calculated by equation (19). The elastic strain of Ag changes from \( +\varepsilon_{\text{Ag},y} \) at 77 K(1) to \( -\varepsilon_{\text{Ag},y} \) at T5.

Step 9 (T5 → RT(3))

During further heating of the laminated tape from T5 (277 K) to RT(3), Ag deforms plastically in compression. The change of strain of each constituent from T5 to RT(3), \( \Delta \varepsilon_{\text{f}, \text{t}5-\text{RT}(3)} \) (\( i = \text{Bi2223, Ag alloy and stainless steel} \)), is calculated by equation (20). The elastic strain of Ag remains at \( -\varepsilon_{\text{Ag},y} \).

Step 10 (RT(3) → T6)

When the laminated tape is cooled down again to 77 K, tensile stress is thermally imposed on Ag. Thus Ag behaves elastically until T6 at which Ag comes to be yielded in tension. The temperature T6, satisfying \( (\alpha_{\text{LT}} - \alpha_{\text{Ag}})(T6 - \text{RT}) = 2\varepsilon_{\text{Ag},y} \), was calculated to be 93 K. The change of strain of each constituent from RT(3) to T6, \( \Delta \varepsilon_{\text{f}, \text{t}RT(3)-\text{t}6} \) (\( i = \text{Bi2223, Ag, Ag alloy and stainless steel} \)), is calculated by equation (19). The elastic strain of Ag changes from \( -\varepsilon_{\text{Ag},y} \) at RT(3) to \( +\varepsilon_{\text{Ag},y} \) at T6.

Step 11 (T6 → 77 K(2))

During further cooling of the laminated tape from T6 (93 K) to 77 K, Ag deforms plastically in tension. The change of strain of each constituent from T6 to 77 K, \( \Delta \varepsilon_{\text{f}, \text{t}6-\text{77 K}} \) (\( i = \text{Bi2223, Ag alloy and stainless steel} \)), is calculated by equation (20). The elastic strain of Ag remains at \( +\varepsilon_{\text{Ag},y} \).

Step 12 (77 K(2) → T7)

At 77 K, Ag is yielded in tension. When the laminated tape is heated from 77 K, compressive stress is thermally imposed on Ag. Thus Ag behaves elastically up to T7 (277 K) at which Ag is yielded in compression, as in step 8. The change of strain of each constituent from 77 K to T7 is the same as \( \Delta \varepsilon_{\text{f}, \text{t}77 \text{K}-\text{t}7} \) (\( i = \text{Bi2223, Ag, Ag alloy and stainless steel} \)) in step 8.

Step 13 (77 K(2) → RT(4))

During further heating of the laminated tape from T7 ( equality T5 = 177 K) to RT(4), Ag deforms plastically in compression, as in step 9. Therefore, the change of strain of each constituent from T7 to RT(4) is the same as \( \Delta \varepsilon_{\text{f}, \text{t}77 \text{K}-\text{RT}(4)} \) (\( i = \text{Bi2223, Ag alloy and stainless steel} \)) in step 9. The elastic strain of Ag remains at \( -\varepsilon_{\text{Ag},y} \).

As stated above, the strain of each component varies in each step in the thermo-mechanical process. The equations used for calculation of the change in elastic strain of Ag (\( \Delta \varepsilon_{\text{Ag}} \)), Ag alloy (\( \Delta \varepsilon_{\text{Alloy}} \)), Bi2223 (\( \Delta \varepsilon_{\text{Bi}} \)) and stainless steel (\( \Delta \varepsilon_{\text{SS}} \)) in the thermo-mechanical process are tabulated in table 2.

### 3.3. Estimation of effectively acting pre-load on stainless steel filament

Figure 4 shows the change of strain \( \varepsilon_{\text{Bi}} \) of Bi2223 filaments in the laminated composite tape at room temperature RT(3) under applied tensile strain \( \varepsilon_{x,T} \), measured by x-ray diffraction. From this result, the residual strain \( \varepsilon_{\text{Bi(RT)}} \) of Bi2223 at room temperature, corresponding to \( \varepsilon_{x} = 0\% \), was found to be \(-0.133\% \). The mechanical strain and the thermally induced strain of each constituent are calculated by the procedure stated above. The total strain of each constituent except Ag is the sum of the mechanical and thermal strains. The total strain of the Bi2223 was measured with the x-ray method at room temperature for the laminated tape with the thermal history TH → T0 → T1 → RT(1) → T2 → T3 → T4 → RT(2) → 77 K → T5 → RT(3) and mechanical history at the lamination temperature T3 (figure 3).

The thermal strain \( \varepsilon_{\text{Bi(T)}} \) (RT(3)) of Bi2223 at RT(3) in the laminated tape with the thermal history mentioned above was...
Diffraction was discussed in Section 3.1, the mechanical strain of Bi2223, $\varepsilon_{\text{Bi}}$, before and during the lamination process. Figure 4 illustrates the change of strain of Bi2223 filaments in the laminated composite, $\varepsilon_{\text{Bi,RT}}$, as a function of the applied tensile strain on composite tape $\varepsilon_{c,T}$. The table below summarizes the equations used for calculation of the change in elastic strain of Ag ($\Delta \varepsilon_{\text{Ag}}$), Ag alloy ($\Delta \varepsilon_{\text{Alloy}}$), Bi2223 ($\Delta \varepsilon_{\text{Bi}}$), and stainless steel ($\Delta \varepsilon_{\text{SS}}$) in the thermo-mechanical process.

### Table 2: Equations used for calculation of the change in elastic strain of Ag, Ag alloy, Bi2223, and stainless steel

<table>
<thead>
<tr>
<th>Process</th>
<th>$\Delta \varepsilon_{\text{Ag}}$</th>
<th>$\Delta \varepsilon_{\text{Alloy}}$</th>
<th>$\Delta \varepsilon_{\text{Bi}}$</th>
<th>$\Delta \varepsilon_{\text{SS}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>$T_0 \rightarrow T_1$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Ag}}) (T_1-T_0)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Alloy}}) (T_1-T_0)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Bi}}) (T_1-T_0)$</td>
</tr>
<tr>
<td>after lamination</td>
<td>$T_1 \rightarrow RT(1)$</td>
<td>$-\Delta \varepsilon_{\text{Ag}}$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Alloy}}) (RT-T_1)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Bi}}) (RT-T_1)$</td>
</tr>
<tr>
<td>T1 to T2</td>
<td>$\Delta \varepsilon_{\text{Ag}}$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Alloy}}) (T_2-T_{\text{RT}})$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Bi}}) (T_2-T_{\text{RT}})$</td>
<td></td>
</tr>
<tr>
<td>T2 to T3, 0</td>
<td>$-\Delta \varepsilon_{\text{Ag}}$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Alloy}}) (T_3-T_2)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Bi}}) (T_3-T_2)$</td>
<td></td>
</tr>
<tr>
<td>Mechanical process</td>
<td>$T_3, 0 \rightarrow T_3, 1$</td>
<td>$F_{\text{IT}}^0 / (E_{\text{LT}} A_{\text{LT}})$</td>
<td>$F_{\text{IT}}^0 / (E_{\text{LT}} A_{\text{LT}})$</td>
<td>$F_{\text{SS}}^0 / (E_{\text{SS}} A_{\text{SS}})$</td>
</tr>
<tr>
<td>during lamination</td>
<td>$T_4$</td>
<td>$\Delta \varepsilon_{\text{Ag}} + \Delta \varepsilon_{\text{Alloy}}$</td>
<td>$\Delta \varepsilon_{\text{Bi}} + \Delta \varepsilon_{\text{SS}}$</td>
<td>$\Delta \varepsilon_{\text{Alloy}} + \Delta \varepsilon_{\text{SS}}$</td>
</tr>
<tr>
<td>T1</td>
<td>$\Delta \varepsilon_{\text{Ag}} + \Delta \varepsilon_{\text{Alloy}}$</td>
<td>$\Delta \varepsilon_{\text{Bi}} + \Delta \varepsilon_{\text{SS}}$</td>
<td>$\Delta \varepsilon_{\text{Alloy}} + \Delta \varepsilon_{\text{SS}}$</td>
<td></td>
</tr>
<tr>
<td>Thermal</td>
<td>$T_3, 3 \rightarrow RT(2)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Ag}}) (RT-T_3)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Alloy}}) (RT-T_3)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Bi}}) (RT-T_3)$</td>
</tr>
<tr>
<td>after lamination</td>
<td>$T_4 \rightarrow T_5$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Ag}}) (T_4-T_3)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Alloy}}) (T_4-T_3)$</td>
<td>$(\alpha_{\text{LT}} - \alpha_{\text{Bi}}) (T_4-T_3)$</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{\text{SS}}$</td>
<td>$T_7$</td>
<td>$-\Delta \varepsilon_{\text{SS}}$</td>
<td>$-\Delta \varepsilon_{\text{SS}}$</td>
<td>$-\Delta \varepsilon_{\text{SS}}$</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{\text{Bi}}$</td>
<td>$T_8$</td>
<td>$-\Delta \varepsilon_{\text{Bi}}$</td>
<td>$-\Delta \varepsilon_{\text{Bi}}$</td>
<td>$-\Delta \varepsilon_{\text{Bi}}$</td>
</tr>
</tbody>
</table>

Figure 4 shows the change of strain of Bi2223 filaments in the laminated composite tape at room temperature (RT3) under applied tensile strain $\varepsilon_{c,T}$, measured by x-ray diffraction. The changes of elastic strain of Bi2223, Ag, Ag alloy, and stainless steel in the thermo-mechanical process were read to 71.4 N. Otto [1] has mentioned a value in the range of 45–98 N for $F_{\text{SS}}^0$. The present result, $F_{\text{SS}}^0 = 71.4$ N, is in such a range. The mechanical strain $\varepsilon_{\text{Bi,M}}$ of Bi2223 induced by the lamination, followed by load relaxation, for $F_{\text{IT}}^0 = 4.5$ N and $F_{\text{SS}}^0 = 71.4$ N was calculated to be $-0.058\%$.

In this way, the comparison of the residual strain of Bi2223 measured by x-rays with the calculation result, the contributions of thermal and mechanical strains to the total residual strain at room temperature RT3 were estimated to be $-0.075$ and $-0.058\%$, respectively.

### 3.4. Change of residual strain of each constituent as a function of temperature

The changes of elastic strain of Bi2223, Ag, Ag alloy and stainless steel in the thermo-mechanical history were calculated by adding the strain change in the order step 1 (T0 $\rightarrow$ T1), step 2 (T1 $\rightarrow$ RT1), step 3 (RT1 $\rightarrow$ T2), step 4 (T2 $\rightarrow$ T3, 0), stage 1 (T3, 0 $\rightarrow$ T3, 1), stage 2 (T3, 1 $\rightarrow$ T4), stage 3 (T4 $\rightarrow$ RT3), and stage 4 (RT3 $\rightarrow$ RT4).
Figure 6. Change of elastic component of strain of Ag in the thermo-mechanical history.

Figure 7. Change of strain of Ag alloy in the thermo-mechanical history.

T3, 2), stage 3 (T3, 2 → T3, 3), step 5 (T3, 3 → T4), step 6 (T4 → RT(2)), step 7 (RT(2) → 77 K(1)), step 8 (77 K(1) → T5), step 9 (T5 → RT(3)), step 10 (RT(3) → T6), step 11 (T6 → 77 K(2)), step 12 (77 K(2) → T5) and step 13 (T5 → RT(4)). The strain of the stainless steel was calculated by adding the strain change from stage 0 (T3, 0) to step 13 (T5 → RT). For calculation, the estimated value of $F_{0SS}^0 = 71.4$ N, $T_0 = 563$ K [27], the values listed in table 1 and equations listed in table 2 were used. The calculation results for Ag, Ag alloy, stainless steel and Bi2223 are presented in figures 6–9, respectively. The following features are seen.

(A) Ag (figure 6) deforms elastically and plastically in tension and compression, depending on the thermo-mechanical history, due to the low yield strain (0.02%). Such a variation of the strain state of Ag affects on the variation of the strain of other constituents as shown below. It is noted that, when the laminated composite tape is thermally cycled between RT and 77 K, the elastic strain of Ag is cycled along RT(3) → T6 → 77 K(2) → T7 → RT(4) → RT(3).

The elastic strain of Ag at RT during thermal cycling is always $-\varepsilon_{Ag,Y}$. On the other hand, in the case where the tape is cooled from the heat-treatment temperature (TH) to RT, the elastic strain of Ag at RT(1) is $+\varepsilon_{Ag,Y}$. In this way, the elastic strain of Ag is different even at the same temperature, being dependent on the thermal history.

(B) The tensile strain is accumulated in Ag alloy in the insert tape with decreasing temperature (figure 7). At room temperature RT(1), the strain of Ag alloy becomes 0.100% when cooled from the heat-treatment temperature. When the tape is heated to the lamination temperature ($T_3 = 453$ K), it decreases to 0.052%. At lamination, the strain becomes 0.058% at T3, 1 due to the applied tensile pre-load (4.5 N). Then the strain decreases to $-0.006\%$ at T3, 3 due to the load relaxation. During cooling from the lamination temperature, tensile strain is also added due to the higher coefficient of thermal expansion than Bi2223 and stainless steel. The strain becomes 0.026% at room temperature (RT(2)) and 0.076% at 77 K(1). It is noted that, when the sample is cooled down to 77 K(1) and then is heated to RT(3), the strain 0.032% at RT(3).
is different from 0.026% at RT(2). This is attributed to the difference in the state of Ag (yielded in tension and compression for the former and latter cases, respectively). When the tape is thermally cycled between RT and 77 K, the strain hysteresis of Ag alloy (and also Bi2223 and stainless steel) becomes cyclic (77 K(1) → T5 → RT(3) → T6 → 77 K(2)), due to the compressive and tensile yielding of Ag at T5 and T6, respectively (figure 6).

(C) The stainless steel (figure 8) is pulled in tension by 0.119% for lamination, and this decreases to 0.056% by the load relaxation after lamination. When the laminated tape is cooled down from the lamination temperature (T3), tensile stress is imposed and the strain at 77 K becomes 0.098%, which is similar to the applied strain 0.119% for lamination.

(D) The strain of Bi2223 (figure 9) is practically accumulated below the temperature T0 due to the difference in coefficient of thermal expansion among the constituents. Then the compressive strain increases with decreasing temperature. At room temperature RT(1), the strain of Bi2223 in the insert tape before lamination is −0.064%. When the tape is heated to the lamination temperature T3, 0 (=453 K), it becomes −0.015%. In advance of the lamination, it is pulled in tension, due to which the strain of Bi2223 becomes −0.009% at T3, 1. In the load relaxation process after lamination, Ag deforms first elastically and then plastically in compression. Accordingly, the strain of Bi2223 varies first from T3, 1 to T3, 2 and then T3, 2 to T3, 3. It becomes −0.073% at T3, 3 after the load relaxation process. When the laminated tape is cooled, the compressive strain increases. At room temperature RT(2), the strain of Bi2223 becomes −0.139%. When the tape is cooled down to 77 K(1) and then heated to room temperature RT(3), the strain of Bi2223 becomes −0.133%, which is the value measured by x-ray diffraction. In this way, the residual strain varies due to the additional thermal cycle between room temperature and 77 K. The reason for this stems from the difference in stress state of Ag (Ag is yielded in tension and compression in the former and latter cases, respectively).

Figures 10 and 11 show the comparison of the variation of the strain in the laminated tape with that in the insert tape alone for Ag alloy and Bi2223, respectively. In the case of bare insert tape, the residual strains of Ag alloy and Bi2223 vary along ABCDEFGHEFG (figure 10). In the case of laminated tape, they vary along ABCDIJKLNOFPQROPQ (figure 11). Three distinct features are found. (1) The residual strains of Ag alloy at room temperature and 77 K in the laminated tape are lower than those in the insert tape. The stainless steel lamination acts to reduce the residual strain of Ag alloy. (2) The compressive residual strains of Bi2223 at room temperature and 77 K in the laminated tape are higher than those in the insert tape. The stainless steel lamination acts to enhance the compressive residual strain of Bi2223, contributing the high tensile strain tolerance of critical current of the laminated tape. (3) The strain hysteresis of Bi2223 (and Ag alloy also) in the thermal cycling between room temperature and 77 K in the laminated tape is smaller than that in the insert tape. The lamination of the insert tape with high modulus stainless steel acts to hide the influence of the change in stress state Ag on the strain of Bi2223 (Ag alloy), reducing the hysteresis.

3.5. Effect of lamination on the strain tolerance of the critical current at 77 K

Figure 12 shows the measured changes of the normalized critical current $I_c/I_{c0}$ at 77 K with applied tensile strain $\varepsilon_{\text{LT}}$ in the insert and laminated tapes, where $I_c$ is the critical current at arbitrary applied tensile strain and $I_{c0}$ is the original current at $\varepsilon_{\text{LT}} = 0$. Based on the result shown in figure 12, the effect of lamination on the strain tolerance of critical current is to be discussed in this subsection.

3.5.1. Improvement of tensile strain tolerance of critical current by lamination. We denote the applied tensile strain on the tape in the early stage of fracture process, at which the critical current is reduced by 5% from the original value (=tensile strain at $I_c/I_{c0} = 0.95$), as $\varepsilon_{T,77 K,I_{c0},95\%_\text{LT}}$ and $\varepsilon_{T,77 K,I_{c0},95\%_\text{LT}}$ for insert and laminated tapes, respectively. The measured values of $\varepsilon_{T,77 K,I_{c0},95\%_\text{LT}}$ and $\varepsilon_{T,77 K,I_{c0},95\%_\text{LT}}$...
3.5.2. Improvement of strain tolerance window of critical current by lamination. Otto et al [1] have shown that the strain tolerance window $\Delta \varepsilon_{w95\%c}$, defined as the axial strain range from compressive through tensile in which the critical current $I_c$ remains above 95% of its original level, is increased by lamination with high modulus reinforcing materials such as stainless steel, invar and molybdenum. They have derived an empirical equation for the strain tolerance window in the form [1]

$$\Delta \varepsilon_{w95\%c} = 0.0036 f_c E_c + 0.40\%,$$  

(29)

where $f_c$ and $E_c$ are the area fraction and Young’s modulus (GPa) of the reinforcement, respectively. The present insert and stainless steel-laminated tapes correspond to $f_c E_c = 0$ and 47.9 GPa, respectively.

Denoting the compressive fracture strain of Bi2223 filaments in the insert tape as $\varepsilon_{Bi,f,C,77 K,IT}$ and that in the laminated tape as $\varepsilon_{Bi,f,C,77 K,LT}$, the strains $\Delta \varepsilon_{w95\%c}$ for the insert and laminated tapes are expressed by

$$\Delta \varepsilon_{w95\%c,IT} = \varepsilon_{Bi,f,C,77 K,IT} - \varepsilon_{Bi,f,C,77 K,LT} - (\varepsilon_{Bi,f,C,77 K,IT} - \varepsilon_{Bi,f,C,77 K,LT}) = \varepsilon_{Bi,f,T,77 K,IT} - \varepsilon_{Bi,f,T,77 K,LT} - \varepsilon_{Bi,f,C,77 K,LT} - \varepsilon_{Bi,f,C,77 K,LT}$$  

(30)

$$\Delta \varepsilon_{w95\%c,LT} = \varepsilon_{Bi,f,T,77 K,LT} - \varepsilon_{Bi,f,T,77 K,LT} - (\varepsilon_{Bi,f,C,77 K,LT} - \varepsilon_{Bi,f,C,77 K,LT}) = \varepsilon_{Bi,f,T,77 K,LT} - \varepsilon_{Bi,f,T,77 K,LT} - \varepsilon_{Bi,f,C,77 K,LT} - \varepsilon_{Bi,f,C,77 K,LT}$$  

(31)

Equations (30) and (31) contain no residual strain term. As shown in 3.5.1, the increase in compressive residual strain acts to raise the strain tolerance of critical current under applied tensile strain. On the other hand, it acts to reduce the strain tolerance under applied compressive strain. Namely, the positive contribution of the increased compressive residual strain under applied tensile strain is counterbalanced with the negative contribution of compressive residual strain under applied compressive strain. Thus, the experimentally observed increase in strain tolerance window in the laminated tape cannot be accounted for by the increase in compressive residual strain. Another mechanism is needed for explanation.

The strain windows $\Delta \varepsilon_{w95\%c,IT}$ and $\Delta \varepsilon_{w95\%c,LT}$ for the insert and laminated tapes are calculated to be 0.40 and 0.57% by equation (29), respectively. Substituting $\Delta \varepsilon_{w95\%c,LT} = 0.40\%$ and $\varepsilon_{Bi,f,T,77 K,LT} = 0.11\%$, we have $\varepsilon_{Bi,f,T,77 K,IT} = 0.29\%$ into equation (30), we have $\varepsilon_{Bi,f,T,77 K,IT} = 0.29\%$ for the insert tape, and substituting $\Delta \varepsilon_{w95\%c,LT} = 0.57\%$ and $\varepsilon_{Bi,f,T,77 K,LT} = 0.22\%$ into equation (31), we have $\varepsilon_{Bi,f,T,77 K,LT} = 0.35\%$ for the laminate tap. This result indicates that the laminate raises the tensile fracture strain of Bi2223 filaments from 0.11 to 0.22% and compressive fracture strain from −0.29 to −0.35%. In other words, the laminate acts to suppress the fracture of the Bi2223 filaments under both applied tensile and compressive strains.

Concerning the increase in tensile and compressive fracture strains of the Bi2223 filaments at $I_c/I_{LO} = 0.95$ between the insert and laminate tapes is considered to be related to the increase in strain tolerance window by lamination reported by Otto et al [1], as follows.
by lamination, the mechanism is unknown within the present work, but the following mechanism might be mentioned from the fracture mechanical viewpoint of fiber-reinforced metal matrix composites [30, 31]. When the weaker Bi2223 filaments are fractured, a stress concentration is induced in the neighboring filaments. Comparing the stress concentration in the intact filament neighboring the fractured one in the insert tape with that in the laminated tape, the latter can be higher than the former due to the existence of the stainless steel, with high modulus. Accordingly, the fracture of the neighboring filaments in the laminated tape tends to be suppressed. On the other hand, the fracture of the weaker filaments tends to induce the fracture of the neighboring filaments in the insert tape. Once the neighboring filaments are fractured in the insert tape, much higher stress concentration is induced on the next neighboring filaments, which causes the fracture of the next neighboring filaments. Due to such a difference in fracture mode of the filaments between the insert and laminated tapes, the greater the number of filaments that the transport superconducting current can survive through in the laminated tape. Such a mechanism could account for the experimentally observed increase in the strain window by lamination from the viewpoint that the lamination acts to change the fracture mode so as to retard the reduction in critical current. Such a mechanism is, however, only one candidate. Further study is needed for full understanding.

4. Conclusions

(1) To describe the thermally induced axial strain changes of the constituents (Bi2223, Ag, Ag alloy and stainless steel) during heating and cooling and mechanically induced strain changes due to the lamination, followed by the stress relaxation, a calculation procedure based on the elastic–plastic mechanics was presented.

(2) The variation of the strain of each constituent in the lamination and subsequent stress relaxation process and in the heating and cooling process in a stainless steel-laminated Bi2223/Ag/Ag alloy superconducting composite tape, fabricated at American Superconductor Corporation, was calculated by inputting the mechanical property values of the constituents and the residual strain of Bi2223 in the laminated composite at room temperature measured by the X-ray diffraction method. It was shown that the stainless steel lamination acts to reduce the residual strain of Ag alloy, to enhance the compressive residual strain of Bi2223 and to retard the fracture of the filaments, contributing to the high tensile strain tolerance of critical current of the laminated tape. Also it was shown that the lamination acts to reduce the strain hysteresis of Bi2223 and Ag alloy in the thermal cycling between room temperature and 77 K.

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