Modeling analysis of distribution of irreversible bending strain for critical current in Bi2223-composite tape


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ABSTRACT

It was attempted to estimate the distribution of irreversible bending strain for critical current of the Bi2223 composite tape. In the modeling, the shape of the core composed of the current transporting Bi2223 filaments and Ag, and the damage strain parameter given by the difference between the intrinsic tensile damage strain and residual strain of the Bi2223 filaments, were incorporated to describe the damage evolution in the core and its relation to critical current. The application of the model to the experimental results of critical current-bending strain relation for many test specimens revealed that the distribution of the irreversible bending strain is described by the two parameter Weibull distribution function. Based on discussion about the influence of the core geometry on the irreversible strain and the result of analysis of the experimental data, a simple and practically useful method was proposed for estimation of the distribution of irreversible bending strain.

1. Introduction

The critical current of Bi2223 composite tape decreases seriously beyond an irreversible strain due to the damage evolution [1–13]. The damage evolution behavior is different from specimen to specimen [3–7], resulting in distribution of irreversible strain among the specimens. The estimation method of such a distribution has, however, not been developed until now. The present work aimed to estimate the distribution of irreversible bending strain among the test specimens, based on the modeling analysis of the experimental critical current-bending strain data.

Fig. 1a shows an optical micrograph of the transverse cross-section of the VAM1 composite tape used in the round robin test [6,7], carried out by the VAMAS (Versailles project on advanced materials and standard)/TWA 16 (Technical working area 16, superconducting materials). The composite tape is composed of the outer sheath of the Ag alloy and the core in which the superconducting current-transporting Bi2223 filaments are embedded in silver. The damage evolution and its relation to the critical current of bent composite tape will be modeled in 2.1 and 2.2 by combining the shape of the core with the damage strain parameter, which refers to the difference between the intrinsic tensile damage strain and residual strain of the Bi2223 filaments.

In the round robin test, the critical current-bending strain curves have been measured for the VAM1 composite tape. Fig. 2 shows the reported change of the normalized critical current \(I_c/I_{c0}\) with increasing bending strain \(e_B\) for 33 tested specimens [7], where \(I_c\) is the critical current at \(e_B = 0\), \(0.2\), \(0.4\), \(0.6\), \(0.8\) and \(1.0\%\) and \(I_{c0}\) is the original value at \(e_B = 0\%.\) The data shown in Fig. 2 will be analyzed in 3.1 and 3.2 by the modeling approach presented in 2.1 and 2.2, and the distribution of the bending irreversible strain \(e_{B,\text{irr}}\) among the 33 specimens will be estimated.

Based on the result in 3.2 and the discussion about the influence of the core geometry on the irreversible strain, a simple and practically useful procedure will be proposed in 3.3, for estimation of the irreversible bending strain distribution from the experimental critical current-bending strain data.

2. Modeling

2.1. Shape of the core

The shape of the core of the VAM1 composite tape (Fig. 1a) used in the round robin test [6,7] was modeled as shown in Fig. 1b (Model R where the shape of the core was approximated to be
and then e first at y. Fig. 3. (a) Optical micrograph, (b) Model R and (c) Model S of the transverse cross-section of the VAM1 composite tape.

Model R. AB : y = 0.0918 for -1.76 \leq x \leq 1.76.
BC : x = 1.76 for -0.0918 \leq y \leq 0.0918.
CD : y = -0.0918 for -1.76 \leq x \leq 1.76 and
DA : x = -1.76 for -0.0918 \leq y \leq 0.0918.

Model S. abc : y = 0.117324 + 1.13901x + 10.0985x^2 + 38.6006x^3 + 83.9271x^4 + 113.805x^5 + 97.9306x^6 + 51.8601x^7 + 15.3722x^8 + 1.94706x^9.

rectangular) and in Fig. 1c (Model S where the practical shape was incorporated). Fig. 3 shows the schematic representation of the geometry of the cross-section in Models R and S. The width- and thickness directions of the composite tape were taken as x and y, respectively, and the center of the composite tape as x = y = 0. The thickness and width of the composite tape were given by t = 0.27 mm for VAM1 sample and W_{sc} (3.52 mm), respectively.

From the observation of the cross-section (Fig. 1a), the shape of the core in Model S (abhcd in Fig. 3) has been formulated in our preceding work \[13\] by 9 order polynomial, respectively, and the center of the composite tape as x = y = 0. The thickness and width of the composite tape were given by t (=0.27 mm for VAM1 sample) and W_{sc} (3.52 mm), respectively.

The normalized critical current \(I_c/I_{c0}\) to bending strain \(\varepsilon_{B}\) is calculated by

\[
e = (\varepsilon_{B}/(\gamma/2)) + \varepsilon_{f}
\]

Noting the intrinsic fracture strain of the filaments as \(\varepsilon_{f}\) and the damage front as \(y_{i}\), and substituting \(e = \varepsilon_{B}\) and \(y = y_{f}\) into Eq. (3), we have \(y_{f}\) as a function of \(\varepsilon_{B}\):

\[
y_{f} = (\varepsilon_{B} - \varepsilon_{f})/(\varepsilon_{B}/(\gamma/2))
\]

The first damage takes place at \(y_{f} = y_{max}(y_{max}(R) and y_{max}(S) for Models R and S, respectively (Fig. 3)), when the bending strain \(\varepsilon_{B}\) reaches the irreversible strain \(\varepsilon_{B,irr}\). Substituting \(y_{f} = y_{max}\) and \(\varepsilon_{B} = \varepsilon_{B,irr}\) into Eq. (4), we have

\[
\varepsilon_{B,irr} = (\gamma_{i} - \gamma_{f})/(y_{max}(\gamma/2))
\]

The \(\gamma_{i} - \gamma_{f}\) value in Eqs. (4) and (5) corresponds to the applied strain at which the filament is damaged in the tensile test [10,13]. It is, hereafter, noted as the damage strain parameter. The normalized critical current \(I_c/I_{c0}\) is 1(unity) until \(\varepsilon_{B}\) reaches \(\varepsilon_{B,irr}\) at which \(y_{i}\) is equal to \(y_{max}\). When \(\varepsilon_{B}(>\varepsilon_{B,irr})\) is raised from \(\varepsilon_{B,irr}\) to \(\varepsilon_{B,irr+1}\) (i correspond to 0.4, 0.6 and 0.8% for the test condition in the round robin test whose result has been presented in Fig. 2), the damage front \(y_{f}\) moves from \(y_{f}\) to \(y_{f+1}\) towards the neutral axis (y = 0), as shown in Fig. 3. The damage extension leads to reduction in cross-sectional area of the current transporting Bi2223 filaments and therefore critical current. The \(I_c/I_{c0}\) for \(\varepsilon_{B} \geq \varepsilon_{B,irr}\) is calculated by

\[
I_c/I_{c0} = S_{surviving}/S_{overall} = 1 - \int_{W_{sc}/2}^{W_{sc}/2} (y - y_{i}) dx/(2W_{sc} \cdot y_{max}(R))
\]
3. Results and discussion

3.1. Application of Models R and S to the relation of normalized critical current $I_{c}/I_{c0}$ to bending strain $\varepsilon_{B}$

Fig. 4 shows the changes of $I_{c}/I_{c0}$ as a function of $\varepsilon_{B}$ for the assumed values of $\varepsilon_{r} - \varepsilon_{c} = 0.2, 0.3$ and 0.4%, calculated for Models R and S. The calculated irreversible bending strain ($\varepsilon_{B,irr}$) values are indicated by arrows. Following features are read from Fig. 4:

1. The damage strain parameter $\varepsilon_{r} - \varepsilon_{c}$ plays a dominant role in determination of the $I_{c}/I_{c0} - \varepsilon_{B}$ curve. The $I_{c}/I_{c0} - \varepsilon_{B}$ curve shifts to the higher bending strain range for higher $\varepsilon_{r} - \varepsilon_{c}$ value in both Models R and S.
2. The $I_{c}/I_{c0} - \varepsilon_{B}$ relation just beyond the irreversible strain shows downward concave in Model R but upward one in Model S.
3. The $I_{c}/I_{c0} - \varepsilon_{B}$ curve in the high $I_{c}/I_{c0}$ range ($I_{c}/I_{c0} > 0.8$) is different between Models R and S. Model R, in which the shape of the core is approximated to be rectangular, gives overestimation for irreversible bending strain. The difference in irreversible strain between Models R and S increases with increasing $\varepsilon_{r} - \varepsilon_{c}$.
4. While the $I_{c}/I_{c0} - \varepsilon_{B}$ curve for $I_{c}/I_{c0} > 0.8$ is different between Models R and S, it is similar in both Models for $I_{c}/I_{c0} < 0.8$. This result suggests that Model R can be used for the description of $I_{c}/I_{c0} - \varepsilon_{B}$ curve for $I_{c}/I_{c0} < 0.8$ as well as Model S.

For both Models R and S, if the value of the damage strain parameter $(\varepsilon_{r} - \varepsilon_{c})$ is known in advance, $I_{c}/I_{c0} - \varepsilon_{B}$ curve can be calculated, as shown above. This, in turn, means that when $I_{c}/I_{c0} - \varepsilon_{B}$ curve is known, $\varepsilon_{r} - \varepsilon_{c}$ can be estimated by the reverse calculation. Once $\varepsilon_{r} - \varepsilon_{c}$ value is known, the irreversible bending strain $\varepsilon_{B,irr}$ can be estimated by Eq. (5). Such an approach will be applied to the experimental result of the round robin test (Fig. 2), as follows.

The results shown in Figs. 4 and 5 suggest that the data in $I_{c}/I_{c0} < 0.8$ are available for estimation of $\varepsilon_{r} - \varepsilon_{c}$ for Model R, while the data in $I_{c}/I_{c0} < 1.0$ are available for Model S. Accordingly, for Model R, the $\varepsilon_{r} - \varepsilon_{c}$ value of each specimen was estimated by fitting Eq. (8) to the measured $I_{c}/I_{c0} - \varepsilon_{B}$ relation for the $\varepsilon_{B}$ range from 0.6% to 1.0% satisfying $I_{c}/I_{c0} < 0.8$. For Model S, it was estimated by fitting Eq. (6) to the measured $I_{c}/I_{c0} - \varepsilon_{B}$ relation for the $\varepsilon_{B}$ range from 0.4% to 1.0% satisfying $I_{c}/I_{c0} < 1.0$. The estimated $\varepsilon_{r} - \varepsilon_{c}$ value was substituted into Eq. (5) to obtain the irreversible bending strain $\varepsilon_{B,irr}$. By repeating such a procedure for all tested specimens, the distribution of $\varepsilon_{B,irr}$ were obtained.

Fig. 6 shows the obtained distribution of $\varepsilon_{B,irr}$ in Models S(a) and R(b). Model R leads to higher $\varepsilon_{B,irr}$. It has been reported that the distribution of $\varepsilon_{B,irr}$ follows the two parameter Weibull distribution function [4]. The $\varepsilon_{B,irr}$ is proportional to $\varepsilon_{r} - \varepsilon_{c}$ (Eq. (5)). Thus it is suggested that the distribution of $\varepsilon_{B,irr}$ also follows the two parameter Weibull distribution. In this function, the cumulative probability $F$ of $\varepsilon_{B,irr}$ is expressed by

![Fig. 4. Change of normalized critical current as a function of bending strain $\varepsilon_{B}$ for $\varepsilon_{r} - \varepsilon_{c} = 0.2, 0.3$ and 0.4%, calculated by Models S and R. The arrows show the irreversible bending strain, $\varepsilon_{B,irr}$.

![Fig. 5. Measured and analyzed change of average critical current ($I_{c}/I_{c0})_{ave}$ with bending strain $\varepsilon_{B}$. The results analyzed by Modes S and R with $\varepsilon_{r} - \varepsilon_{c} = 0.25\%$ are shown with solid and broken curves, respectively. The average irreversible bending strains analyzed by Models S (0.29%) and R (0.37%) are also presented.]
When the lnln(1–e) and converting the cumulative probability m respectively. Substituting the estimated for Model S, respectively, and to be 6.41% and 0.392% for Model R, the distribution of e respectively. Fig. 7 shows the result of the Weibull plot. High solid curves in Fig. 6. The distribution of e for the results of Models S and R. The solid curves show the distribution calculated with the shape and scale parameters estimated from the Weibull plot (Fig. 7).

\[ F = 1 - \exp\left(-\frac{(e_\text{B,irr}}{e_0})^m\right) \]  

(9)

where \( e_0 \) and m are the scale and shape parameters, respectively. Eq. (9) is re-written in the form: \( \ln[\ln(1-F)^{-1}] = m \ln(e_\text{B,irr}) - m \ln(e_0) \). When the \( \ln[\ln(1-F)^{-1}] \) is plotted against \( \ln(e_\text{B,irr}) \) (Weibull plot), the m- and \( e_0 \)-values can be estimated from the slope and extrapolation, respectively. Fig. 7 shows the result of the Weibull plot. High linearity between \( \ln[\ln(1-F)^{-1}] \) and \( \ln(e_\text{B,irr}) \) is found, indicating that the distribution of \( e_\text{B,irr} \) obeys the two parameter Weibull distribution. The m and \( e_0 \) were estimated to be 7.23% and 0.309% for Model S, respectively, and to be 6.41% and 0.392% for Model R, respectively. Substituting the estimated m- and \( e_0 \)-values in Eq. (9) and converting the cumulative probability F to the frequency (probability density) \( f \), we had the \( f-e_\text{B,irr} \) diagram, as shown with solid curves in Fig. 6. The distribution of \( e_\text{B,irr} \) is well described both for the results of Models S and R.

3.3. Conversion of the distribution of \( e_\text{B,irr} \) estimated by Model R to that by Model S

As Model S is based on the practical shape of the core, it can give accurate irreversible strain distribution. However, it needs time for calculation. On the other hand, Model R is very simple and needs not time for calculation. However, Model R gives overestimation for irreversible strain (Fig. 6). In this subsection, the relation of distribution of \( e_\text{B,irr} \) estimated by Model R to that by Model S is studied, based on which a simple procedure to estimate the distribution of irreversible strain from the results of Model R will be presented.

The irreversible bending strain \( e_\text{B,irr} \) is given by Eq. (5). As the \( e - e_\text{r} \) value estimated by Model R for \( l/l_0 < 0.8 \), corresponding to the bending strain range 0.6–1.0%, is almost the same as that estimated by Model S, it can be used as a common value for Models R and S for a given specimen. Under a given \( e - e_\text{r} \) value, the difference in \( e_\text{B,irr} \) between Models R and S is related to the difference in \( y_{\text{max}(R)} \) and \( y_{\text{max}(S)} \) for Models R and S respectively (Fig. 3). Noting the values of \( e_\text{B,irr} \) estimated by Models R and S as \( e_\text{B,irr}(R) \) and \( e_\text{B,irr}(S) \), respectively, and applying Eq. (5), we have

\[ e_\text{B,irr}(S) = \left(\frac{y_{\text{max}(R)}}{y_{\text{max}(S)}}\right) e_\text{B,irr}(R) \]  

(10)

As \( y_{\text{max}(R)} \) and \( y_{\text{max}(S)} \) have been measured experimentally to be 0.0918 and 0.117 mm, the \( e_\text{B,irr}(R) \) can be converted to \( e_\text{B,irr}(S) \) by Eq. (10). The conversion factor, \( y_{\text{max}(R)}/y_{\text{max}(S)} \), is 0.785. The \( e_\text{B,irr}(R) \) value of each specimen estimated by Model S shown in Fig. 6b was converted by Eq. (10) with the conversion factor 0.785. The Weibull plot of such converted values \( e_\text{B,irr}(R \rightarrow S) \) is presented in Fig. 7, which is nearly the same as that estimated by Model S. From the slope and extrapolation of the plot in Fig. 7, the shape and scale parameters for the converted values were estimated to be 6.41 (same as that of the result of Model R) and 0.308, respectively. Fig. 8 shows the comparison of the distribution of \( e_\text{B,irr}(R \rightarrow S) \) with that of \( e_\text{B,irr}(S) \) and \( e_\text{B,irr}(R) \). The distribution of the \( e_\text{B,irr}(R \rightarrow S) \) values is almost the same as that of \( e_\text{B,irr}(S) \). Such a result suggests that the \( e_\text{B,irr} \) values estimated by Model R are available for estimation of distribution of \( e_\text{B,irr} \) by using the conversion factor (Eq. (10)).

Based the result above, as a practically useful tool for estimation of distribution of \( e_\text{B,irr} \), the method with a simple procedure “measurement of \( y_{\text{max}(S)} \) and \( y_{\text{max}(R)} \) and estimation of the conversion factor \( y_{\text{max}(R)}/y_{\text{max}(S)} \)”, “estimation of \( e - e_\text{r} \) by Eq. (8) and then \( e_\text{B,irr} \) by Eq. (5) based on Model (R)” is proposed, which gives almost the same result as Model S.

Fig. 6. Distribution of the irreversible bending strain \( e_\text{B,irr} \) estimated by Models S and R. The solid curves show the distribution calculated with the shape and scale parameters estimated from the Weibull plot (Fig. 7).

Fig. 7. Weibull plot of irreversible bending strain \( e_\text{B,irr} \). S, R and R → S refer to the \( e_\text{B,irr} \) values estimated by Model S, Model R and conversion from Model R to S, respectively.

Fig. 8. Distribution of irreversible bending strain \( e_\text{B,irr} \) estimated by Models S, Model R and conversion from Model R to S.
4. Conclusions

(1) A model for description of the distribution of irreversible bending strain of Bi2223 composite tape was presented, in which the shape of the core and the parameter damage strain parameter were incorporated for description of the damage evolution and its relation to critical current.

(2) The model was applied to the reported data of the round robin test carried by VAMAS/TWA16. The distribution of the irreversible strain was described by the two parameter Weibull distribution.

(3) A simple and practically useful method was proposed for estimation of the distribution of irreversible bending strain, based on the discussion about the influence of the core geometry on the irreversible strain and the result of the analysis of the experimental data by the present model.

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